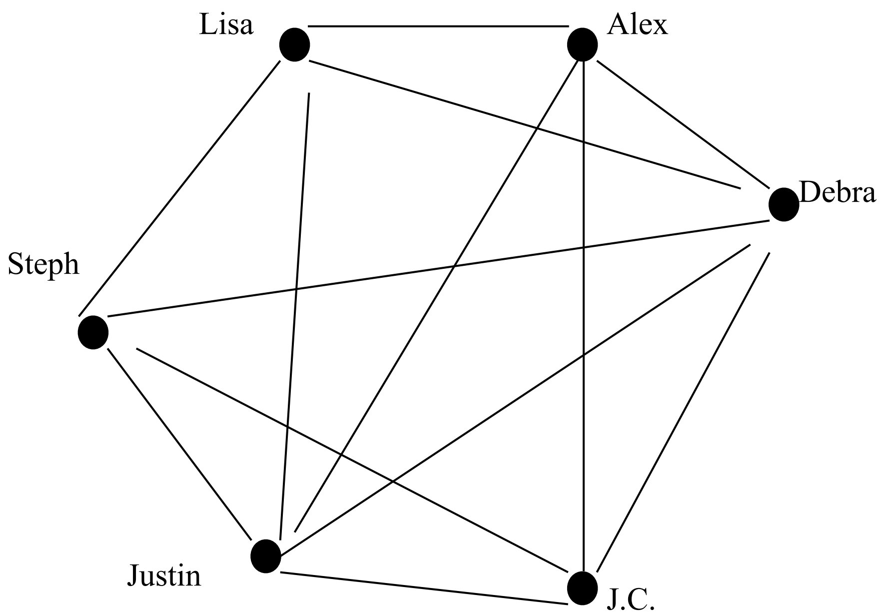
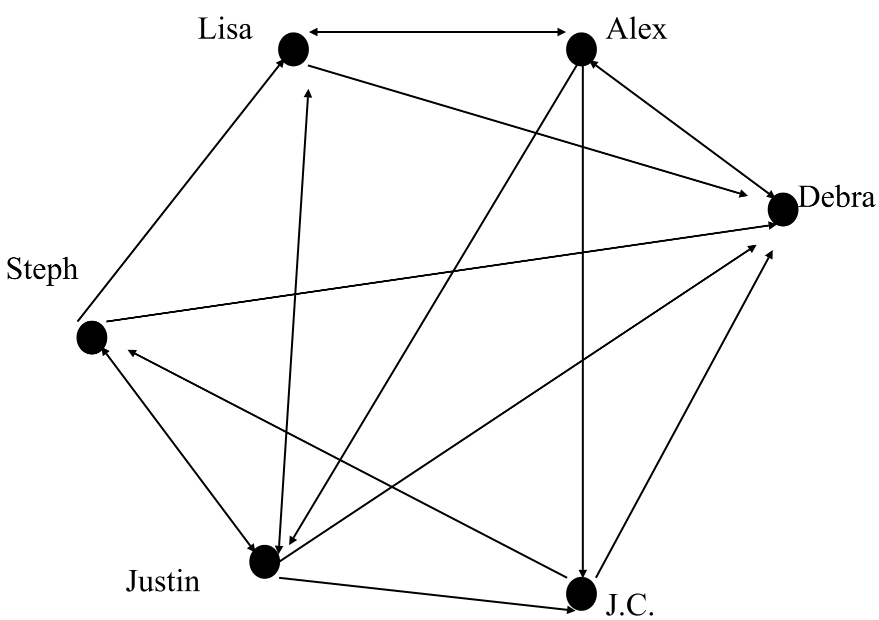
Social Networks and Discrete Mathematics

Social Networks are an engaging topic for students to investigate when considering networks. Consider the social network below. This can represent friends at school, Facebook "friends", a Google+ Circle, or Twitter "Tweeters".



Arrows can be used in networks to indicate one-way emails, posts on social network site "walls" or blogs, gossiping, or even to represent borrowing and loaning of money. In that case, a directed graph would provide insight into the social network.

The network below shows the interactions between 6 Facebook Friends. An arrow toward a person means that person received a post on their wall. An arrow away from a person means that person wrote the post. Arrows on both ends means the two people posted on each other's walls and therefore received a post from each other.



Adjacency matrices help analyze networks. In order to do so, you must define what values of 1 and 0, horizontal, and vertical means.

A value of 1 in any row means that the row person posts to the column person's wall.
A value of 0 means they do not post on the column person's wall.
A value of 1 in any column means they receive a post from the row person.
A value of 0 in any column means they do not receive a post from that row person.
**Please note, it is generally accepted that each person posts to their own wall in Facebook.

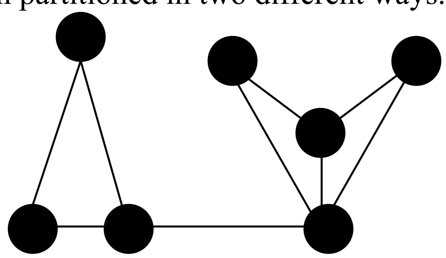
	Lisa	Steph	Justin	J.C.	Debra	Alex
Lisa	1	0	1	0	1	1
Steph	1	1	1	0	1	0
Justin	1	1	1	1	1	0
J.C.	0	1	0	1	1	0
Debra	0	0	0	0	1	1
Alex	1	0	1	1	1	1

Using the matrix we are able to understand the interactions better.

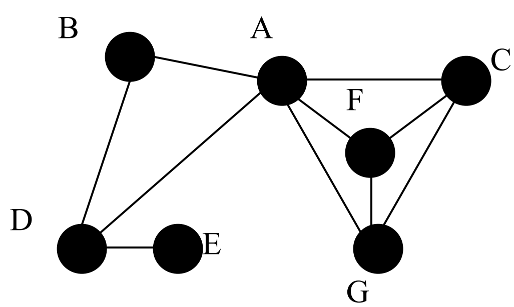
For example, who is the most "outgoing" or "outspoken/written"?
Who is the "most popular"?
Who is the least popular?
Who is the most "introverted"?

If we redefine an arrow with two tips to indicate a BFs, Do all of the friends have a BF? Who has the most? Who has the least? (**Note, a new matrix may be helpful, or looking at this matrix in a different way may help.)

Sometimes, we look at networks as trees. It can be helpful to divide a network into *natural* communities. This can highlight the circles in a Google+ account and whom they have in common. It can highlight cliques within a larger group. For example, if the network were a marching band, the natural communities would show the different cliques or circles of friends within the band. This partitioning is useful in the workplace as well. Partitioning can help identify working groups that already exist in a workplace, and can also highlight the crucial employees that are part of multiple natural communities. These employees are usually the last to be considered for layoffs or down-sizing.
The same graph is shown partitioned in two different ways.



By partitioning the network, we can understand whether there are small groups within a group and whether there are members that are peripheral members because they have very limited interaction within the network. Consider the example below of a workplace network.

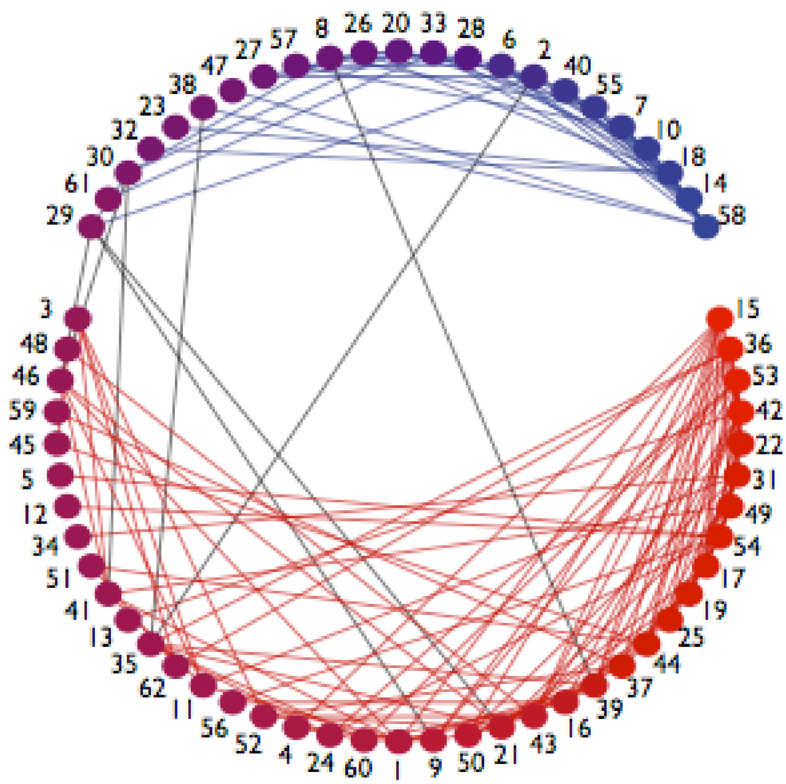


There are clearly two different working groups in the network. A-B-D-E is a work group and A-C-F-G is a work group. Each node has a degree of two or more except E. Therefore, E is a peripheral member of the network. E has limited interaction since E's only access to the network is through D. By analyzing the degrees of each node we can also determine that community A-C-F-G is a stronger community than A-B-D-E because all members of A-C-F-G have a degree of 3, a greater degree than most members of A-B-D-E.

Identifying communities within a network and considering the degrees of each node will help students understand the network better and be able to consider:

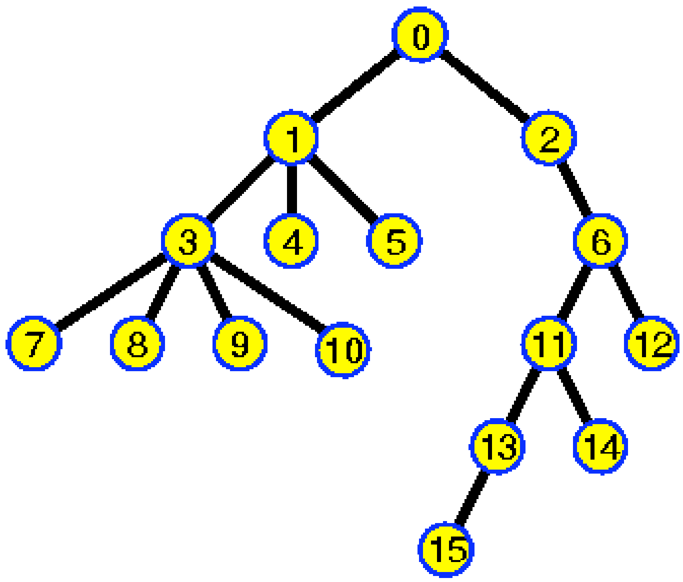
- the strength of the community
- which nodes belong in which communities
- where communities exist—or if they do not exist
- whether any members are weakly involved in the network
- and can even help determine the impact of layoffs or downsizing operations.

Partitioning can occur in much more complicated networks as well, like the biological network below.



Another consideration when analyzing networks is what can cause them to fail? When analyzing a network for failures, it becomes helpful to draw the tree representation. First let’s review some tree basics.

TREE DEFINITIONS

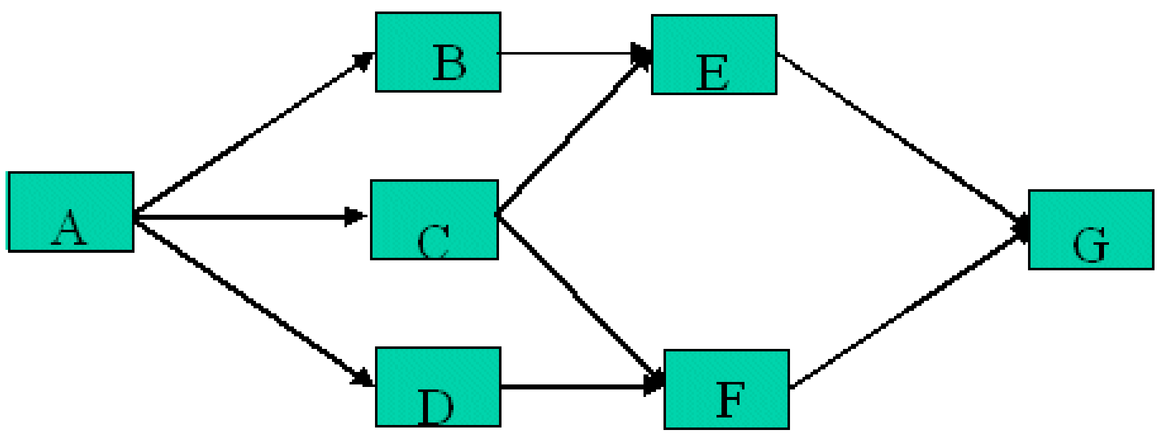


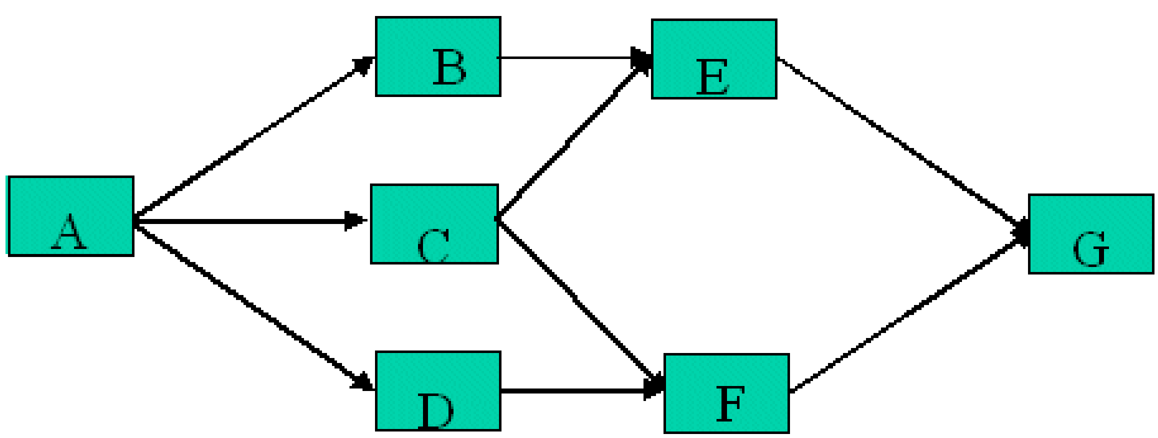
Tree has 16 nodes
Tree has degree 4
Tree has depth 5
Node 0 is the root
Node 1 is internal
Node 4 is a leaf
4 is a child of 1
1 is the parent of 4
0 is grandparent of 4
3, 4 and 5 are siblings

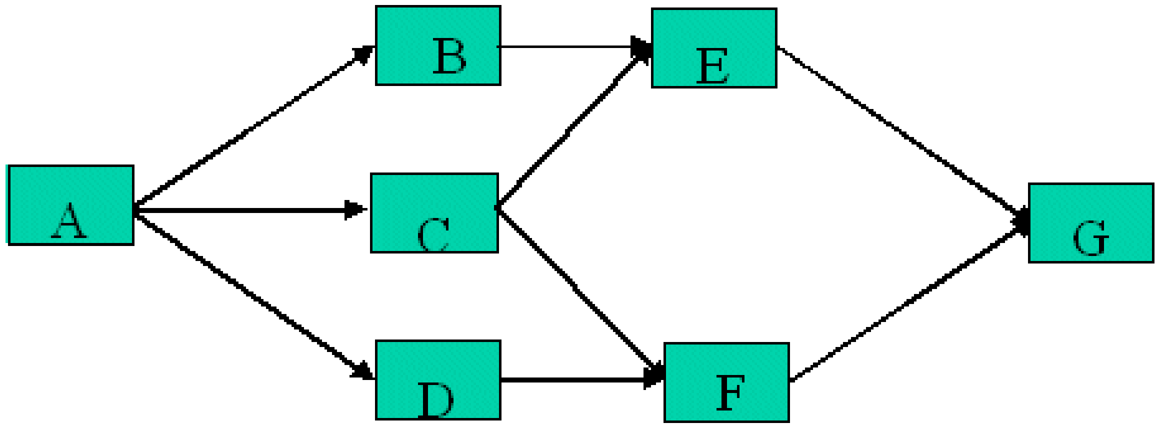
Why analyze a tree? There is a wealth of information contained in a tree. If the tree is a telecommunications network, then we can consider cut sets as the nodes that must fail in order for the network to fail. By understanding this, we can try to build a safer network and we can try to safeguard the network that has already been built.
In the tree above, if node 0 fails the network fails. If 1 and 2 fail the network fails. If 1 and 6 fail the network fails. If 1, 11, and 12 fail, then the network will also fail. If 3, 4, 5, and 2 fail then the network fails. There are many cut sets that can be made. Once those are outlined we can consider the minimum cut set. That is, the minimum nodes that can fail for the network to fail?

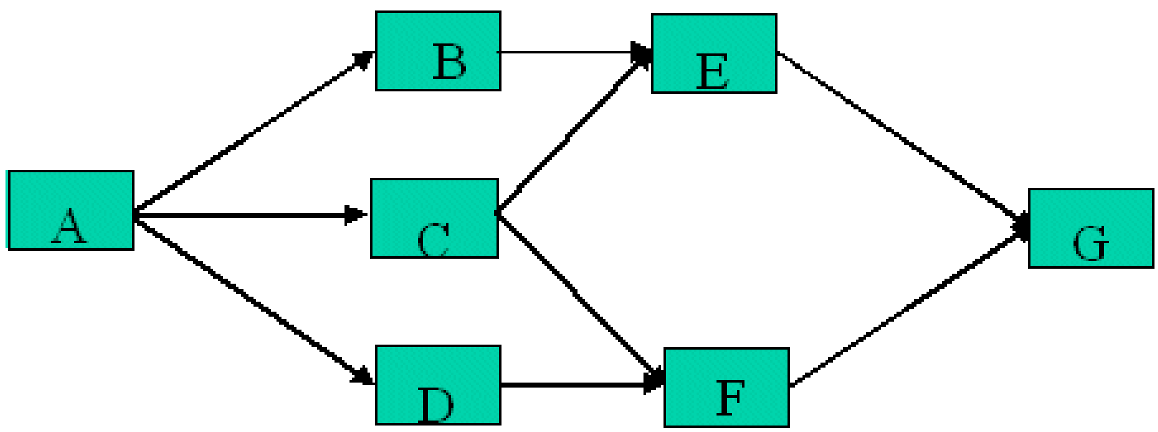
How does this relate to social networks? Considering the social network as a system, what are the minimum losses of friends for the social network to fail? What are the minimum nodes that can fail before a community fails? The minimum nodes that fail each community will be a cut set, but will not always be the minimum cut set.

Consider the social network below. It was developed by tracking a joke that was originally posted by A. The joke was then posted by B,C,D at roughly the same time. E and F posted the joke. Finally G posted the joke. There was a set deadline for looking for the joke, to allow us to stop tracking the network (for simplicity). Once the tree is built, questions can help analyze the network. Are there *natural* communities? Are they easy or difficult to see in this representation? Are there cut sets that would have "killed the joke"? Do you see any other options?
Use the network below to partition the tree in different ways to identify natural communities and cut sets.






A
B
C
D
E
F
G


A
B
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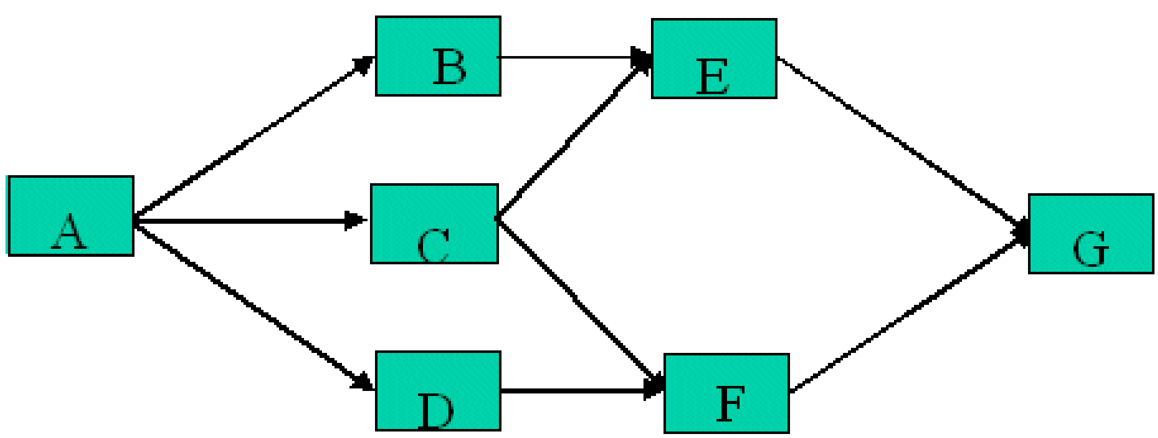
G


A
B
C
D
E
F
G

Which partition makes the least sense? Which one makes the most sense? Why?

Did some partitions appear to use equivalent rationale for partitioning?

Choose a joke (it must be funny) and begin posting it on one student's account. Track the joke and create your own tree to analyze. This can be a very interesting activity if multiple social networking sites are used. One person can post it on Google+, one person can post it on Facebook, and one person can tweet it. Analyze the trees and compare them. What do they say about each social networking site's structure? clientele? etc.

There are many other possibilities to investigate when analyzing networks, but these represent a springboard for discussions and investigations.

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